

# Specification and estimation of a dynamic consumption allocation model

December 2012

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**Abstract** - This paper presents an allocation model of Belgian household consumption over 23 categories of goods and services. We have formulated and estimated an extension of the classic Almost Ideal Demand System. The original model has been modified by introducing a dynamic adjustment mechanism and by the inclusion of demographic variables. These capture shifts in consumption patterns related to the changing age composition of the population. The paper is an extension of earlier work (Willemé, 2008), in particular with respect to the number and composition of the consumption categories considered.

**Abstract** – Deze paper stelt een allocatiemodule voor van de consumptie van de Belgische gezinnen over 23 categorieën van goederen en diensten. We hebben een uitbreiding van het klassieke Almost Ideal Demand System geformuleerd en geschat. Het originele model is uitgebreid met de invoering van een dynamisch aanpassingsmechanisme en de toevoeging van demografische variabelen. Deze variabelen vangen verschuivingen in de consumptiepatronen op die het gevolg zijn van veranderingen in de leeftijdsstructuur van de bevolking. Deze paper is een uitbreiding van vroeger werk (Willemé, 2008), meer bepaald wat betreft het aantal consumptiecategorieën en de allocatiestructuur.

**Abstract** – Ce Working Paper présente un nouveau module d'allocation de la consommation privée des ménages belges, divisant celle-ci en 23 catégories de biens et services. Pour ce faire, nous avons formulé et estimé une extension du système AIDS (Almost Ideal Demand System). Le modèle original a été modifié en introduisant un mécanisme d'ajustement dynamique et des variables démographiques. Ces

variables prennent en compte des glissements dans la consommation privée suite à des changements dans la structure d'âge de la population. Ce papier est une extension des travaux antérieurs (Willemé, 2008), notamment concernant le nombre de catégories et la structure d'allocation.

**Jel Classification** – B23, C22, C51, D10

**Keywords** - Consumption allocation model, AIDS

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## Executive summary

This paper presents an allocation model of Belgian household consumption over 23 categories of goods and services. We have formulated and estimated an extension of the classic Almost Ideal Demand System. The original model has been modified by introducing a dynamic adjustment mechanism and by the inclusion of demographic variables. These capture shifts in consumption patterns related to the changing age composition of the population. The paper is an extension of earlier work (Willemé, 2008), in particular with respect to the number and composition of the consumption categories considered.

The allocation module presented in this Working Paper has the advantage that all private consumption expenditures are represented in one allocation structure. The consumption categories rent and private health expenditure are not estimated separately as in the allocation module presented in Working Paper 5-04. Private health expenditure could be included thanks to the inclusion of demographic variables.

Implied long-run overall income and uncompensated price elasticities are calculated for the whole allocation structure by simulating the allocation module alone up to horizon 2100 keeping prices, total consumption expenditure and demographic variables constant. The adjustment process brings the system to a steady state. Next a 1 percent increase in total real consumption expenditure or in individual prices of consumption categories is introduced in period 2050. In the absence of further shocks the error correction mechanism will pull the system to a new steady state. In period 2050 the deviations from the baseline can be interpreted as short-run income and uncompensated price elasticities, while the responses in period 2059 approximate the long-run elasticities. These 'equilibrium' elasticities obtained by simulating the allocation module alone differ from 'real world' elasticities because there are no feedback mechanisms.

The implied long-run income elasticities are relatively high for recreation, other goods and services, clothing, personal transport equipment and energy consumption categories. Communication, consumption abroad, domestic services, medical care, food, non-alcoholic and alcoholic beverages, tobacco, rent and purchased transport have rather low long-run income elasticities.

The long-run uncompensated own price elasticities, which take into account the income effect of a price change, are relatively low for communication, consumption abroad, domestic services, medical care, food, non-alcoholic beverages, rent (positive sign for but non-significantly different from zero), furniture, purchased transport and energy consumption categories. Alcoholic beverages, tobacco, clothing and personal transport equipment show rather high price elasticities.

# 1. Introduction

In this paper we specify an allocation model of private expenditures, embedded in a generalisation of Deaton & Muellbauer's (1980) Almost Ideal Demand System (AIDS). The main advantage of specifying a complete demand system is that this is the only way to take into account the fact that all household consumption decisions are subject to the same budget restriction. By implication, every determinant of spending on any consumption is a potential determinant of spending on all or some other goods and services.

The paper is organized as follows. Section 2 presents the specification of the model. It is identical to section 3 in Willemé (2008) and is repeated here for convenience. Taking the original AIDS specification as a starting point, the model is extended by introducing a dynamic adjustment mechanism and by the inclusion of demographic variables. These were expected to capture shifts in consumption patterns related to the changing age composition of the population. Section 3 provides a brief overview of the data used to estimate the model. The estimation results, more precisely the implied income and price elasticities, are discussed in section 4. Section 5 concludes.



## 2. Specification of the consumption allocation model

### 2.1. The static AIDS model

One of the most widely used consumption allocation models in empirical research is the Almost Ideal Demand System (AIDS) (Deaton & Muellbauer, 1980). Its popularity stems from its generality (combining appealing features of both the Rotterdam and the translog models) and the ease with which (restrictions on) its parameters can be estimated. The demand system is specified in terms of a set of budget share equations of the following form (all variables measured in current period, observation subscripts dropped for clarity):

$$w_i = \alpha_i + \sum_j \gamma_{ij} \ln P_j + \beta_i \ln \left( \frac{X}{P} \right), \quad i=1, \dots, n; j=1, \dots, n \quad (1)$$

where

$$\ln P = \alpha_0 + \sum_i \alpha_i \ln P_i + 0.5 \sum_i \sum_j \gamma_{ij} \ln P_i \ln P_j \quad i=1, \dots, n; j=1, \dots, n \quad (2)$$

$w_i$  = the  $i$ -th budget share

$P_i$  = the price of the  $i$ -th commodity

$X$  = nominal income <sup>1</sup>.

Since the budget shares sum to unity by definition, the model parameters must satisfy the following adding-up restrictions:

$$\sum_i \alpha_i = 1 \quad \sum_i \beta_i = 0 \quad \sum_i \gamma_{ij} = 0 \quad i=1, \dots, n \quad (3)$$

Imposing these restrictions results in one equation of the system to become redundant, implying that one equation may be dropped for estimation. Uncompensated price and income elasticities can be derived from (1) and (2) fairly easily. They are listed below:

$$\varepsilon_{ij} = \frac{1}{w_i} \left[ \gamma_{ij} - \beta_i \left( \alpha_j + 0.5 \sum_{j=1}^n (\gamma_{ij} + \gamma_{ji}) \ln P_j \right) \right] - \delta_{ij}, \quad \delta_{ij} = 1 \text{ if } i=j, 0 \text{ otherwise} \quad (4)$$

<sup>1</sup> It is customary to refer to  $X$  as income, although the variable actually represents total expenditures on all items, i.e. disposable household income minus saving.

$$\eta_i = 1 + \frac{\beta_i}{w_i} \quad (5)$$

It should be noted that these elasticities are not constant, since they depend on the evolution of income and prices.

The theory of the rational consumer implies that demand functions are homogeneous of order zero, ensuring that a proportional change in all prices and nominal income does not affect the quantities demanded. Price effects are also expected to be symmetric. The implications of these restrictions on the elasticities are discussed in the next sections.

### 2.1.1. Homogeneity restrictions

Imposing homogeneity conditions on model (1) implies the following restrictions on the model parameters:

$$\sum_j \gamma_{ij} = 0 \quad j = 1, \dots, n \quad (6)$$

Equations (1) and (2) change accordingly, and the price elasticities now become:

$$\varepsilon_{ij} = \frac{1}{w_i} \left[ \gamma_{ij} - \beta_i \left( \alpha_j + 0.5 \sum_{j=1}^{n-1} (\gamma_{ij} + \gamma_{ji}) \ln \left( \frac{P_j}{P_n} \right) \right) \right] - \delta_{ij} \quad (7)$$

$$\delta_{ij} = 1 \text{ if } i = j, 0 \text{ otherwise}$$

The income elasticities remain unchanged.

### 2.1.2. Symmetry restrictions

Symmetry of the price effects can be imposed by setting  $\gamma_{ij} = \gamma_{ji}$ . The uncompensated price elasticities then simplify to:

$$\varepsilon_{ij} = \frac{1}{w_i} \left[ \gamma_{ij} - \beta_i \left( \alpha_j + \sum_{j=1}^{n-1} \gamma_{ij} \ln \left( \frac{P_j}{P_n} \right) \right) \right] - \delta_{ij} \quad \delta_{ij} = 1 \text{ if } i = j, 0 \text{ otherwise} \quad (8)$$

Note that while the homogeneity restrictions apply to each equation separately, the symmetry restrictions are cross-equation restrictions, which can only be imposed when the share equations (1) are estimated simultaneously.

### 2.1.3. A linear approximation (LA-AIDS)

A disadvantage of system (1) and (2) in empirical modelling is the fact that it is nonlinear in the parameters. Deaton & Muellbauer suggest using a linear approximation to the price index (2), known as Stone's (1953) price index:

$$\ln P = \sum_i w_i \ln P_i \quad i=1, \dots, n \quad (9)$$

The approximation appears to work well when the underlying price variables are highly correlated. The obvious advantage of using (9) is that the price index becomes independent of the model parameters, and the model becomes linear. As a result, the expression for the price elasticities simplifies to:

$$\varepsilon_{ij} = \frac{1}{w_i} (\gamma_{ij} - \beta_i w_j) - \delta_{ij} \quad \delta_{ij} = 1 \text{ if } i=j, 0 \text{ otherwise} \quad (10)$$

It is a well-known fact that most early empirical applications of the static AIDS model described in this paragraph have generally failed to confirm the homogeneity and symmetry restrictions, apparently refuting the underlying theoretical model of the rational consumer. This was also the case in the original application of Deaton & Muellbauer (1980), who used post-war British data to test their model. The authors offered several possible explanations for this failure, two of which have proved to be of great importance: (i) the static model implies that consumers adjust their budget allocation in the current period, while it is quite likely that expenditure on several items is relatively inflexible in the short run, implying a dynamic adjustment mechanism; (ii) the use of a 'representative household' assumption to ensure exact aggregation over households will be unwarranted when the distribution of household budgets and demographic structure change over time. In subsequent work these complications have led to extensions of the original model, which we discuss in the following paragraphs.

## 2.2. The dynamic LA-AIDS model

In their 1980 article, Deaton & Muellbauer suggest to include lagged explanatory variables to capture the sluggishness in the adjustment process to the optimal budget allocation. As a result of subsequent developments in time series methods, however, it has become customary to specify the model in terms of a general error correction model (ECM). This approach allows for short-term dynamic adjustments to suboptimal allocations (which occur because of exogenous shocks to the explanatory variables), while preserving a stable long-run relationship between the structural variables. Assuming that the dynamics of the system can be captured with an autoregressive distributed lag model of order one (ADL(1)), the  $i$ -th budget share equation in ECM-form has the following specification:

$$\Delta w_{i,t} = \alpha_i + \sum_j \gamma_{ij}^D \Delta \ln P_{j,t} + \beta_i^D \Delta \ln \left( \frac{X_t}{P_t} \right) + \lambda_i \left[ w_{i,t-1} - \sum_j \gamma_{ij} \ln P_{j,t-1} - \beta_i \ln \left( \frac{X_{t-1}}{P_{t-1}} \right) \right] \quad (11)$$

where  $\lambda_i$  is the usual error correction adjustment parameter, and the long-term equilibrium relationship can be retrieved from the term in square brackets. Homogeneity and symmetry restrictions can be imposed as before, but it should be noted that the dynamic model allows for these restrictions to hold only in equilibrium (in the levels part of the equations). More concretely, the restrictions may be imposed (and tested) on the  $\gamma_{ij}$  but not on the  $\gamma_{ij}^D$ .

### 2.3. Heterogeneity in the AIDS model

As pointed out in the previous paragraph, Deaton and Muellbauer (1980) already suggested that the heterogeneity of household characteristics may have been one of the reasons behind the failure of the model to confirm the standard assumptions of microeconomic demand theory empirically. Indeed, the simple AIDS model is only a valid aggregation of underlying household demand functions under rather restrictive conditions about the distribution of household characteristics and their interaction with price and income effects (see Blundell, Pashardes & Weber, 1993). This point can be illustrated easily by considering a simple example: suppose that household spending on, for example, health care increases with the average age of its members, and that this average increases over time due to the ageing of the population. As a result, the aggregate share of health care in the budget will increase over time at given relative prices and income distribution. This uptrend in health spending is likely to be captured by the income variable, whose estimated effect will be biased upward.

Many sources of household heterogeneity are conceivable<sup>2</sup>, and it goes without saying that it is quite impossible to incorporate them all in the aggregate consumption model. Past research hints at two possible candidate variables: income inequality and demographic composition (Blundell & Stokes, 2005). The former leads to the inclusion of squared income terms, the latter to the inclusion of variables that measure aspects of household composition such as average size and age. Of these candidates, the age composition of the population appears to be a promising choice: not only is there ample evidence of a strong correlation between age and health care spending (see, e.g., European Economy, 2006), it also seems quite likely that age influences many other consumption decisions<sup>3</sup>.

The simplest way to introduce demographic variables in the AIDS model is in log-additive form:

$$w_i = \alpha_i + \sum_j \gamma_{ij} \ln P_j + \beta_i \ln \left( \frac{X}{P} \right) + \delta_i \ln A \quad (12)$$

with the additional restriction that the  $\delta_i$  parameters sum to zero. The assumption implicit in this specification is that the demographic variable(s) (denoted as  $A$  in (12)) do(es) not influence the income effect  $\beta_i$ . While this assumption is somewhat restrictive, it limits the number of additional parameters to be estimated, a considerable advantage when the model is estimated with annual aggregate data.

The empirical model estimated in the next section is the dynamic (ECM) version of equation (12).

<sup>2</sup> For instance, households (and their members) differ in terms of income, educational level, and employment status, to mention only the most obvious sources of heterogeneity.

<sup>3</sup> Blundell et al. (1993), for example, find significant effects for several age-related variables in their estimated equations for food and alcohol expenditure shares.

### 3. Description of the model data

The data used to estimate the demand system described in section 2 consist of total consumption expenditures, budget shares, price indices and demographic variables over the period 1980-2010.

The data source for private consumption expenditures is the Classification of Individual Consumption According to Purpose (COICOP) which is used for the national accounts SNA95. Observations cover the period from 1995 to 2010. For the period 1980-1995 retropolations are made based on growth rates of the national accounts SNA79.

Before a model like (12) can be estimated, we must decide which aggregate demographic variable(s) to be used and which aggregates of consumption goods and services to consider. The former decision depends on the way in which demographics are assumed to play a role at the micro-level. In the empirical application that we describe in this paragraph, we will use the share of people aged between 65 and 74, and of people 75 years and older in the total population. This corresponds with a micro model in which expenditure equations contain two dummy variables corresponding to each age interval, taking the value of one when a household contains at least one member in the interval.

The decision about the consumption categories depends on the level of aggregation of the available data and on assumptions about the separability between groups of these basic commodities. Grouping is unavoidable when more than a few basic consumption categories are available, because the number of parameters grows with the square of the number of categories. It imposes an assumption of separability of the underlying utility functions, implying that the choice between consumption categories of a group depends only on the relative prices of the consumption categories of the group and on total group expenditures, but not on prices of consumption categories of other groups. The separability requirement naturally leads to the use of factor analysis to identify uncorrelated groups of consumption categories. The results suggested the following hierarchical structure: in a first stage the economic agent decides how much to spend on group 1, group 2 and group 3. In a second stage she decides how much of the amount spent on group 1 will be allocated to group 1A, 1B and 1C. These budgets are further allocated over different consumption categories as shown in Graph 1.

Table 1 presents the “within-group” and the “overall” average budget shares and the standard deviation of the annual growth rates of real consumption expenditures over the period 1980-2010. As we could expect the budget shares of food and rent are highest with an overall budget share for food of 14% on average during the period 1980-2010 and an overall budget share for rent of 15%. The standard deviations indicates that expenditures on energy and particularly on heating grow at a rather volatile rate compared to expenditures on food and rent. Expenditures on personal transport equipment, non-alcoholic beverages, communication, tobacco and medical care grow also at a relatively volatile rate.

Graph 1 The Allocation Structure of Total Consumption Expenditures

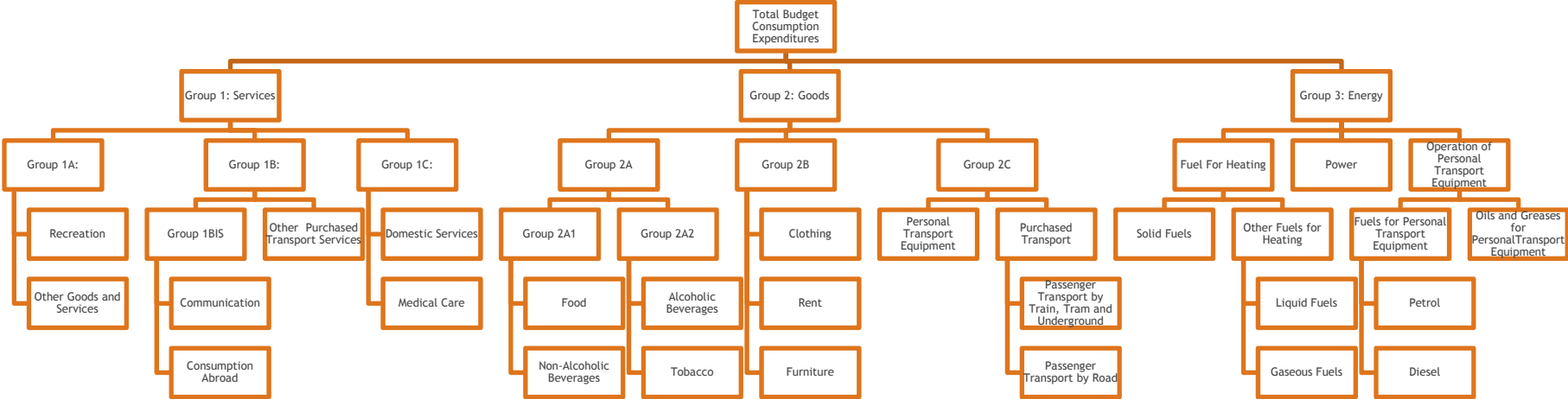
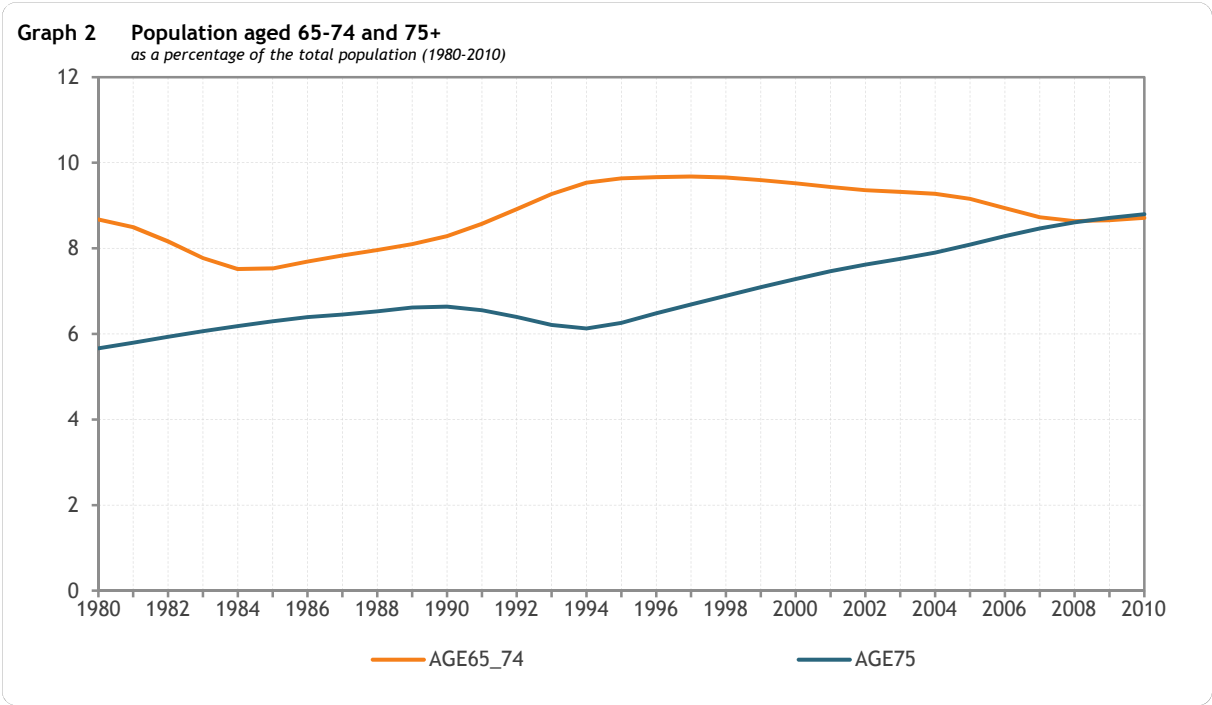


Table 1 Within and Overall Average Budget Shares and Standard Deviation of Annual Growth Rates of Real Expenditure

	Within Average Budget Shares 1980-2010	Standard Deviation of Annual Growth Rates of Real Expenditure 1980-2010	Overall Average Budget Shares 1980-2010
<b>1. Group 1: Services</b>	<b>0.42</b>	<b>1.67</b>	<b>0.424</b>
- <b>Group 1A:</b>	<b>0.72</b>	<b>1.88</b>	<b>0.307</b>
. Recreation	0.28	3.02	0.086
. Other Goods and Services	0.72	2.08	0.220
- <b>Group 1B:</b>	<b>0.15</b>	<b>4.25</b>	<b>0.065</b>
. <b>Group 1B1S:</b>	<b>0.96</b>	<b>4.37</b>	<b>0.062</b>
Communication	0.29	5.13	0.018
Consumption Abroad	0.71	5.49	0.044
. <i>Other Transport Services</i>	<i>0.04</i>	<i>8.21</i>	<i>0.003</i>
- <b>Group 1C:</b>	<b>0.12</b>	<b>4.58</b>	<b>0.052</b>
. Domestic Services	0.14	3.01	0.007
. Medical Care	0.86	5.39	0.044
<b>2. Group 2: Goods</b>	<b>0.50</b>	<b>1.20</b>	<b>0.500</b>
- <b>Group 2A:</b>	<b>0.38</b>	<b>1.48</b>	<b>0.189</b>
. <b>Group 2A1:</b>	<b>0.81</b>	<b>1.75</b>	<b>0.153</b>
Food	0.93	1.80	0.142
Non-Alcoholic Beverages	0.07	5.17	0.011
. <b>Group 2A2:</b>	<b>0.19</b>	<b>2.79</b>	<b>0.036</b>
Alcoholic Beverages	0.47	2.69	0.017
Tobacco	0.53	4.10	0.019
- <b>Group 2B:</b>	<b>0.54</b>	<b>1.27</b>	<b>0.270</b>
. Clothing	0.22	3.21	0.061
. Rent	0.57	1.07	0.152
. Furniture	0.21	2.09	0.057
- <b>Group 2C:</b>	<b>0.08</b>	<b>5.69</b>	<b>0.041</b>
. <i>Personal Transport Equipment</i>	<i>0.80</i>	<i>6.82</i>	<i>0.033</i>
. <i>Purchased Transport</i>	<i>0.20</i>	<i>3.91</i>	<i>0.008</i>
Transport by Train, Tram, Underground	0.55	5.42	0.005
Transport by Road	0.45	4.48	0.004
<b>3. Group 3: Energy</b>	<b>0.08</b>	<b>2.73</b>	<b>0.077</b>
- <b>Fuel for Heating</b>	<b>0.30</b>	<b>6.90</b>	<b>0.024</b>
. <i>Solid Fuels</i>	<i>0.06</i>	<i>8.80</i>	<i>0.001</i>
. <i>Other Fuels for Heating</i>	<i>0.94</i>	<i>6.95</i>	<i>0.022</i>
Liquid Fuels	0.46	7.34	0.011
Gaseous Fuels	0.54	7.76	0.012
- <b>Power</b>	<b>0.30</b>	<b>3.24</b>	<b>0.022</b>
- <b>Operation of Personal Transport Equipment</b>	<b>0.40</b>	<b>2.53</b>	<b>0.031</b>
. <i>Fuels for Personal Transport Equipment</i>	<i>0.90</i>	<i>2.90</i>	<i>0.028</i>
Petrol	0.62	3.66	0.017
Diesel	0.38	3.92	0.010
. <i>Oils and Greases for Personal Transport Equipment</i>	<i>0.10</i>	<i>15.57</i>	<i>0.003</i>

As already mentioned, two demographic variables, which capture the ageing of the population, are defined as the population aged 65 to 74 and the population aged 75 and over, both expressed as a percentage of the total population. Their evolution over the sample period is shown in Graph 2.





## 4. Estimation of the ECM-LA-AIDS model

To estimate the AIDS model described in the previous sections, we have followed a ‘general-to-specific’ modelling strategy as advocated by David Hendry and others (see for example, Charemza & Deadman, 1992). Taking the general dynamic AIDS model as the starting point (the ‘General Unrestricted Model’), we have tested the validity of the following sequence of simplifying restrictions on the model parameters:

- using the linear approximation to the overall price index;
- homogeneity of the demand equations;
- symmetry of the cross-price effects;
- exclusion of non-significant effects.

These restrictions, except the first one, lead to simpler models which are nested in the more general specifications that precede them, so they can be tested with the familiar likelihood ratio (LR) test. The linear price index model, however, cannot be derived from the nonlinear one by imposing restrictions on the latter, nor can both models be conceived as special cases of a common ‘parent’ model, so neither the LR test nor Hausman’s J-test can be used to test the linear approximation. Consequently, either another non-nested formal test should be carried out (such as Vuong’s test, 1989; see Greene 2008), or the validity of the approximation should be judged more informally from the estimated log-likelihoods of the alternative models. A final observation is in order: the linear approximation based on the Stone price index, while being the specification commonly used in applied work, is by no means the only possible one. Indeed, recent work (Ogura, 2006) suggests that the Paassche price index may perform just as well, or possibly even better, than the Stone index.

In this section we summarize the major estimation results in the form of the implied long-run overall income and uncompensated price elasticities (see table 2). In order to obtain these elasticities we first check if the allocation model is well-behaved i.e. we check if it gives stable results when simulating for a long simulation period. For this purpose we simulate the allocation module alone - i.e. not with the complete Hermes model - up to horizon 2100 keeping prices, total consumption expenditure and demographic variables constant. The adjustment process brings the system to a steady state. Next we introduce a 1 percent increase in total real consumption expenditure or in individual prices of consumption categories (period 2050). In the absence of further shocks the error correction mechanism will pull the system to a new steady state. In period 2050 the deviations from the baseline can be interpreted as short-run income and uncompensated price elasticities, while the responses in period 2059 approximate the long-run elasticities. These ‘equilibrium’ elasticities obtained by simulating the allocation module alone differ from ‘real world’ elasticities because of the absence of feedback mechanisms.

**Table 2 Long-run Overall Uncompensated Price and Income Elasticities**

	Group 1: Services	- Group 1A:	.Recreation	.Other Goods and Services	- Group 1B:	.Group 1BIS:	Communication	Consumption Abroad	.Other Transport Services	- Group 1C:	.Domestic Services	.Medical Care	Group 2: Goods	- Group 2A:	.Group 2A1:	Food	Non-Alcoholic Beverages	.Group 2A2:	Alcoholic Beverage	Tobacco	- Group 2B	.Clothing	.Rent	.Furniture	- Group 2C:	.Personal Transport Equipment	.Purchased Transport	Transport Train, Tram, Underground	Transport by Road	Group 3: Energy	- Fuel for Heating	.Solid Fuels	.Other Fuels for Heating	Liquid Fuels	Gaseous Fuels	- Power	- Operation of Personal Transport Eq.	.Fuels for Personal Transport Eq.	Petrol	Diesel	.Oils & Greases Personal. Transport Eq.
Group 1: Services	-0.35	-0.48	-0.48	-0.48	-0.05	-0.05	-0.08	-0.04	-0.05	-0.07	-0.10	-0.06	-0.52	-0.39	-0.40	-0.38	-0.60	-0.38	-0.13	-0.59	-0.52	-1.47	-0.09	-0.93	-0.95	-1.11	-0.08	-0.10	-0.06	-0.75	-0.98	-0.98	-0.98	-0.78	-1.09	-0.69	-0.55	-0.55	-0.55	-0.55	-0.55
- Group 1A:	-0.24	<b>-0.34</b>	-0.34	-0.34	-0.17	-0.17	-0.25	-0.13	-0.17	0.19	0.16	0.20	-0.37	-0.28	-0.28	-0.27	-0.42	-0.27	-0.09	-0.41	-0.36	-1.04	-0.06	-0.65	-0.67	-0.78	-0.06	-0.07	-0.04	-0.53	-0.69	-0.69	-0.69	-0.55	-0.77	-0.49	-0.39	-0.39	-0.39	-0.39	-0.39
.Recreation	-0.07	-0.09	<b>-0.71</b>	0.14	-0.05	-0.05	-0.07	-0.04	-0.05	0.05	0.04	0.05	-0.10	-0.08	-0.08	-0.07	-0.12	-0.07	-0.03	-0.11	-0.10	-0.28	-0.02	-0.18	-0.18	-0.22	-0.01	-0.02	-0.01	-0.14	-0.19	-0.19	-0.19	-0.15	-0.21	-0.13	-0.11	-0.11	-0.11	-0.11	-0.11
.Other Goods and Services	-0.17	-0.24	0.37	<b>-0.48</b>	-0.12	-0.12	-0.18	-0.10	-0.12	0.14	0.12	0.15	-0.27	-0.20	-0.20	-0.19	-0.31	-0.20	-0.07	-0.30	-0.26	-0.75	-0.04	-0.48	-0.49	-0.57	-0.04	-0.05	-0.03	-0.38	-0.50	-0.50	-0.50	-0.40	-0.56	-0.36	-0.28	-0.28	-0.28	-0.28	-0.28
- Group 1B:	-0.06	0.09	0.09	0.09	<b>-0.45</b>	-0.45	-0.71	-0.33	-0.45	-0.34	-0.33	-0.35	-0.09	-0.07	-0.07	-0.06	-0.10	-0.06	-0.02	-0.10	-0.09	-0.25	-0.01	-0.16	-0.16	-0.19	-0.01	-0.02	-0.01	-0.13	-0.17	-0.17	-0.17	-0.13	-0.18	-0.12	-0.09	-0.09	-0.09	-0.09	-0.09
.Group 1BIS:	-0.06	0.08	0.08	0.08	-0.43	<b>-0.47</b>	-0.74	-0.35	0.53	-0.33	-0.31	-0.33	-0.08	-0.06	-0.06	-0.06	-0.10	-0.06	-0.02	-0.10	-0.08	-0.24	-0.01	-0.15	-0.15	-0.18	-0.01	-0.02	-0.01	-0.12	-0.16	-0.16	-0.16	-0.13	-0.18	-0.11	-0.09	-0.09	-0.09	-0.09	-0.09
Communication	-0.02	0.02	0.02	0.02	-0.16	-0.18	<b>-0.03</b>	-0.24	0.13	-0.08	-0.08	-0.08	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.01	-0.02	-0.02	-0.06	0.00	-0.04	-0.04	-0.05	0.00	0.00	0.00	-0.03	-0.04	-0.04	-0.04	-0.03	-0.04	-0.03	-0.02	-0.02	-0.02	-0.02	-0.02
Consumption Abroad	-0.03	0.06	0.06	0.06	-0.26	-0.29	-0.71	<b>-0.10</b>	0.40	-0.25	-0.23	-0.25	-0.06	-0.05	-0.05	-0.05	-0.07	-0.05	-0.02	-0.07	-0.06	-0.18	-0.01	-0.11	-0.12	-0.14	-0.01	-0.01	-0.01	-0.09	-0.12	-0.12	-0.12	-0.10	-0.13	-0.08	-0.07	-0.07	-0.07	-0.07	
.Other Transp. Services	0.00	0.00	0.00	0.00	-0.02	0.02	0.03	0.02	<b>-0.97</b>	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.00	-0.01	-0.01	-0.01	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	
- Group 1C:	-0.05	-0.23	-0.23	-0.23	0.57	0.57	0.89	0.43	0.57	<b>0.08</b>	0.06	0.09	-0.07	-0.05	-0.05	-0.05	-0.08	-0.05	-0.02	-0.08	-0.07	-0.19	-0.01	-0.12	-0.12	-0.14	-0.01	-0.01	-0.01	-0.10	-0.13	-0.13	-0.13	-0.10	-0.14	-0.09	-0.07	-0.07	-0.07	-0.07	
.Domestic Services	-0.01	-0.04	-0.04	-0.04	0.10	0.10	0.15	0.07	0.10	0.02	<b>-0.36</b>	0.09	-0.01	-0.01	-0.01	-0.01	-0.01	0.00	-0.01	-0.01	-0.03	0.00	-0.02	-0.02	-0.02	0.00	0.00	0.00	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.01	-0.01	-0.01	-0.01	-0.01	
.Medical Care	-0.04	-0.19	-0.19	-0.19	0.47	0.47	0.74	0.36	0.47	0.07	<b>0.42</b>	<b>-0.01</b>	-0.06	-0.04	-0.04	-0.04	-0.06	-0.04	-0.01	-0.06	-0.06	-0.16	-0.01	-0.10	-0.10	-0.12	-0.01	-0.01	-0.01	-0.08	-0.10	-0.10	-0.10	-0.08	-0.12	-0.07	-0.06	-0.06	-0.06	-0.06	
Group 2: Goods	-0.97	-1.32	-1.32	-1.32	-0.14	-0.14	-0.22	-0.10	-0.14	-0.19	-0.27	-0.17	<b>0.00</b>	0.07	0.08	0.07	0.15	0.05	-0.26	0.30	0.01	-0.64	0.42	-0.64	-0.33	-0.38	-0.03	0.37	-0.50	-0.16	-0.21	-0.21	-0.21	-0.17	-0.24	-0.16	-0.11	-0.11	-0.11	-0.11	
- Group 2A:	-0.34	-0.47	-0.47	-0.47	-0.05	-0.05	-0.08	-0.04	-0.05	-0.07	-0.09	-0.06	0.01	<b>-0.13</b>	-0.13	-0.12	-0.19	-0.14	-0.32	0.00	0.27	0.78	0.05	0.49	-0.95	-1.11	-0.08	-0.10	-0.06	-0.06	-0.07	-0.07	-0.07	-0.06	-0.08	-0.06	-0.04	-0.04	-0.04	-0.04	
.Group 2A1:	-0.27	-0.37	-0.37	-0.37	-0.04	-0.04	-0.06	-0.03	-0.04	-0.05	-0.07	-0.05	0.01	-0.09	<b>-0.46</b>	-0.44	-0.69	1.21	0.42	1.85	0.22	0.62	0.04	0.39	-0.75	-0.88	-0.06	-0.08	-0.05	-0.04	-0.06	-0.06	-0.06	-0.05	-0.06	-0.04	-0.03	-0.03	-0.03	-0.03	
Food	-0.25	-0.34	-0.34	-0.34	-0.03	-0.03	-0.05	-0.03	-0.03	-0.05	-0.07	-0.04	0.01	-0.08	-0.42	<b>-0.41</b>	-0.54	1.11	0.39	1.70	0.20	0.57	0.03	0.36	-0.69	-0.81	-0.06	-0.07	-0.04	-0.04	-0.05	-0.05	-0.05	-0.04	-0.06	-0.04	-0.03	-0.03	-0.03		
Non-Alcoh. Beverages	-0.02	-0.03	-0.03	-0.03	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.00	0.00	-0.01	-0.04	-0.03	<b>-0.16</b>	0.10	0.03	0.15	0.02	0.05	0.00	0.03	-0.06	-0.07	0.00	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00			
.Group 2A2:	-0.07	-0.10	-0.10	-0.10	-0.01	-0.01	-0.02	-0.01	-0.01	-0.01	-0.02	-0.01	0.00	-0.04	0.33	0.32	0.50	<b>-1.34</b>	-0.75	-1.82	0.06	0.16	0.01	0.10	-0.20	-0.23	-0.02	-0.02	-0.01	-0.01	-0.02	-0.02	-0.02	-0.01	-0.02	-0.01	-0.01	-0.01	-0.01		
Alcoholic Beverage	-0.03	-0.04	-0.04	-0.04	0.00	0.00	-0.01	0.00	0.00	-0.01	-0.01	-0.01	0.00	-0.02	0.15	0.14	0.22	-0.60	<b>-0.76</b>	-0.47	0.03	0.07	0.00	0.05	-0.09	-0.10	-0.01	-0.01	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	0.00	0.00	0.00		
Tobacco	-0.04	-0.05	-0.05	-0.05	-0.01	-0.01	-0.01	0.00	-0.01	-0.01	-0.01	-0.01	0.00	-0.02	0.19	0.18	0.28	-0.74	0.01	<b>-1.36</b>	0.03	0.09	0.01	0.06	-0.11	-0.13	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	0.00	0.00	0.00		
- Group 2B:	-0.52	-0.71	-0.71	-0.71	-0.07	-0.07	-0.12	-0.05	-0.07	-0.10	-0.14	-0.09	-0.02	-0.30	-0.30	-0.29	-0.42	-0.29	-0.11	-0.45	<b>-0.20</b>	-1.24	0.39	-1.01	1.88	2.20	0.14	0.18	0.10	-0.09	-0.11	-0.11	-0.11	-0.09	-0.13	-0.09	-0.06	-0.06	-0.06		
.Clothing	-0.10	-0.13	-0.13	-0.13	-0.01	-0.01	-0.02	-0.01	-0.01	-0.02	-0.03	-0.02	0.00	-0.06	-0.06	-0.05	-0.08	-0.06	-0.02	-0.08	-0.04	<b>-0.91</b>	0.18	0.12	0.36	0.42	0.03	0.04	0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.01	-0.01	-0.01			
.Rent	-0.32	-0.43	-0.43	-0.43	-0.04	-0.04	-0.07	-0.03	-0.04	-0.06	-0.09	-0.05	-0.01	-0.18	-0.18	-0.18	-0.26	-0.18	-0.07	-0.27	-0.12	0.25	<b>0.01</b>	0.92	1.15	1.34	0.09	0.11	0.06	-0.05	-0.07	-0.07	-0.07	-0.06	-0.08	-0.05	-0.04	-0.04			
.Furniture	-0.10	-0.14	-0.14	-0.14	-0.01	-0.01	-0.02	-0.01	-0.01	-0.02	-0.03	-0.02	0.00	-0.06	-0.06	-0.06	-0.08	-0.06	-0.02	-0.09	-0.03	-0.58	0.20	<b>-0.22</b>	0.38	0.44	0.03	0.04	0.02	-0.02	-0.02	-0.02	-0.02	-0.02	-0.03	-0.02	-0.01	-0.01			



The implied long-run income elasticities are relatively high for recreation, other goods and services, clothing, personal transport equipment and energy consumption categories. Communication, consumption abroad, domestic services, medical care, food, non-alcoholic and alcoholic beverages, tobacco, rent and purchased transport have rather low long-run income elasticities.

The long-run uncompensated own price elasticities, who take into account the income effect of a price change, are relatively low for communication, consumption abroad, domestic services, medical care, food, non-alcoholic beverages, rent (positive sign for but non-significantly different from zero), furniture, purchased transport and energy consumption categories. Alcoholic beverages, tobacco, clothing and personal transport equipment show rather high price elasticities.

## 5. Conclusion

In this paper we have used an extended version of Deaton and Muellbauer's Almost Ideal Demand System to model the allocation of the household budget over 23 consumption categories using Belgian data over the 1980-2010 period. The original static model was modified in two important ways. It was made dynamic using the familiar error correction formulation of a first-order autoregressive specification, and the usual determinants of the budget allocation decision (the total real budget and relative prices) were appended with demographic variables that capture the age-related heterogeneity of household consumption patterns.

Implied long-run overall income and uncompensated price elasticities are calculated for the whole allocation structure by simulating the allocation module alone up to horizon 2100 keeping prices, total consumption expenditure and demographic variables constant. The adjustment process brings the system to a steady state. Next a 1 percent increase in total real consumption expenditure or in individual prices of consumption categories is introduced in period 2050. In the absence of further shocks the error correction mechanism will pull the system to a new steady state. In period 2050 the deviations from the baseline can be interpreted as short-run income and uncompensated price elasticities, while the responses in period 2059 approximate the long-run elasticities. These 'equilibrium' elasticities obtained by simulating the allocation module alone differ from 'real world' elasticities because there are no feedback mechanisms.

The implied long-run income elasticities are relatively high for recreation, other goods and services, clothing, personal transport equipment and energy consumption categories. Communication, consumption abroad, domestic services, medical care, food, non-alcoholic and alcoholic beverages, tobacco, rent and purchased transport have rather low long-run income elasticities.

The long-run uncompensated own price elasticities, which take into account the income effect of a price change, are relatively low for communication, consumption abroad, domestic services, medical care, food, non-alcoholic beverages, rent (positive sign for but non-significantly different from zero), furniture, purchased transport and energy consumption categories. Alcoholic beverages, tobacco, clothing and personal transport equipment show rather high price elasticities.

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